Tax Instruments and the Achievement of Tax Equity

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Abstract— **Deductions and tax credits are instruments usually adopted by tax systems, in addition to marginal tax rates, to get the desired tax revenue, together with other fiscal policy objectives, respecting vertical and horizontal equity requirements. The effect of marginal tax rates, deductions and tax credits is here decomposed in order to detect whether and to which extent each of these three instruments violate tax equity.**

Keywords— **Equity in personal income tax, Equity violation decomposition, Kakwani-Lambert axioms.**

I. INTRODUCTION

The effects on taxes, due to marginal tax rates, deductions and tax credit, were first analyzed by Pfäler [7]; then the analysis was refined by Lambert [3] and, recently by Onrubia et Al. [5]. The Pfäler-Lambert-Onrubia et al. approach only focus on how the redistributive effect is achieved. This paper explores which features of the tax contribute the most to the removal of a real-world tax from a perfectly equal one. Kakwani and Lambert [2] (henceforth KL) state that tax should increase monotonically with respect to people's ability to pay (Axiom 1); richer people should pay taxes at higher rates (Axiom 2); no re-ranking should occur in people's living standards (Axiom 3). A violation of Axiom 1 automatically entails a violation of Axiom 2, although not necessarily the other way round. Moreover, Axiom 3 can be violated only if Axiom 2 (and consequently Axiom 1) holds. For the sake of simplicity, we limit our analysis to Kakwani and Lambert (henceforth KL) axiom 1 and axiom 3. Our approach is inspired by the works by Podder [8] [9] and Podder and Chatterjee [10]. The goal is to measure the share of the redistributive effect due to each element of the tax, given the application of all the other tax elements, that is to say from an ex-post overall perspective.

II. PRELIMINARY DEFINITIONS

Let X be the distribution of gross (pre-tax) incomes order in non decreasing order. Let D be the distribution of deduction ordered in non decreasing order and $D_{/X}$ the distribution of deduction ranked by X. Analogously we define C and $C_{/X}$ the distribution of tax-credits. Let V (and $V_{/X}$) the distribution which is given by the application of the tax schedule to X, Let S (and $S_{/X}$) the distribution which is given after having applied the tax schedule to $B=X-D$. And finally let's define by Y (and $Y_{/X}$) the distribution of net incomes, i.e. $Y_{/X} = X - S_{/X} + C_{/X} := X - T_{/X}$; T ($T_{/X}$) are net taxes. $\tilde{T}$ ($\tilde{T}_{/X}$) are average net tax-rates.

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The redistributive effect RE, is defined as

$$RE = G_X - G_Y = G_X - C_{Y|X} - R_{Y|X} = \tau K - R_{Y|X} . \qquad (1)$$

In (1) τ is the ratio between the tax average and the average of the after tax income, i.e. $\tau = \mu_T / \mu_Y$; $C_{Y|X}$ is the pseudo Gini (concentration) coefficient of $Y_{/X}$ (i.e. Y ranked by X). $R_Y = \left(G_Y - C_{Y|X} \right)$ is the Atkinson-Plotnick-Kakwani re-ranking index; K is the Kakwany progressivity index, $K = \left(C_{T|X} - G_X \right)$. We recall the definition of the Reynolds-Smolensky index

$$RS = G_X - C_{Y|X} = \tau K , \qquad (2)$$

from which $RE = RS - R_{Y|X}$.

According to KL $AV_1^T = \tau \left(G_T - C_{T|X} \right)$ measures the violation of axiom 1, as it reveals whether taxes are not ranked as gross incomes as they should. $AV_2^T = \tau \left(G_{\tilde{T}} - C_{\tilde{T}|X} \right)$ measures the violation of axiom 2, as it reveals whether average net tax-rates are not ranked as gross incomes as they should. $AV_3^T = G_Y - C_{Y|X}$ measures the violation of axiom 3, as it reveals whether after tax incomes are not ranked as gross incomes as they should. For the sake of simplicity, in the pursue we omit the analysis of axiom 2 violations, as it is quite analogous to the one described for axiom 1.

III. BASIC DECOMPOSITIONS

Following Podder [8], [9] and Podder and Chatterjee [10], $C_{T|X}$ can be decomposed as follows

$$C_{T|X} = \frac{\mu_V}{\mu_T} r_{V|X} G_V - \frac{\mu_{V-S}}{\mu_T} r_{V-S|X} G_{V-S} - \frac{\mu_C}{\mu_T} r_{C|X} G_C , \qquad (3)$$

where $r_{Z|X} = C_{Z|X} / G_Z$, $Z = V, (V-S), C$.

Note that $-1 \le r_{Z|X} \le 1$, as $-G_Z \le C_{Z|X} \le G_Z$.

By making use of (3), the index RS can be represented as follows

$$RS = \frac{\mu_T}{\mu_Y} \left[\frac{\mu_V}{\mu_T} \left(r_{V|X} G_V - G_X \right) \right.$$
$$\left. + \frac{\mu_T}{\mu_Y} \left[-\frac{\mu_{V-S}}{\mu_T} \left(r_{V-S|X} G_{V-S} - G_X \right) - \frac{\mu_C}{\mu_T} \left(r_{C|X} G_C - G_X \right) \right] . (4)$$

We can analogously decompose G_T

$$G_T = \frac{\mu_V}{\mu_T} r_{V|T} G_V - \frac{\mu_{V-S}}{\mu_T} r_{V-S|T} G_{V-S} - \frac{\mu_C}{\mu_T} r_{C|T} G_C . \qquad (4)$$

For what concerns G_Y and $C_{Y|X}$, we have

$$G_Y = \frac{\mu_{X-V}}{\mu_Y} r_{X-V|Y} G_{X-V} - \frac{\mu_{V-S}}{\mu_Y} r_{V-S|Y} G_{V-S} - \frac{\mu_C}{\mu_Y} r_{C|Y} G_C , \qquad (6)$$

$$C_{Y|X} = \frac{\mu_{X-V}}{\mu_Y} r_{X-V|X} G_{X-V} - \frac{\mu_{V-S}}{\mu_Y} r_{V-S|X} G_{V-S} - \frac{\mu_C}{\mu_Y} r_{C|X} G_C, \quad (7)$$

respectively. On the basis of the decompositions (3)-(5), and (6)-(7), we can decompose the contribution to axiom violations due to the marginal tax-rates, deductions and tax credits.

IV. Decomposing Axiom Violations

The KL measure of axiom 1 violation, can be decomposed as

$$AV_1^T = \tau\left(G_T - C_{T|X}\right) = AAV_1^V + AAV_1^D + AAV_1^C, \quad (8)$$

where

$$AAV_1^V = \frac{\mu_V}{\mu_V}\left(r_{V|T} - r_{V|X}\right)G_V, \quad AAV_1^C = -\frac{\mu_C}{\mu_V}\left(r_{C|T} - r_{C|X}\right)G_C,$$

and $AAV_1^D = -\frac{\mu_{V-S}}{\mu_V}\left(r_{V-S|T} - r_{V-S|X}\right)G_{V-S}$.

According to Pellegrino and Vernizzi [6], let's now write

$$C_{Z|E} = \frac{1}{2\mu_Z N^2} \sum_{i=1}^{K} \sum_{j=1}^{K} \left(z_i - z_j\right) p_i p_j I_{i-j}^{Z|E}, \quad I_{i-j}^{Z|E} = \begin{cases} 1: & e_i > e_j \\ -1: & e_i < e_j \\ I_{i-j}^Z: & e_i = e_j \end{cases}$$

$Z=V, (V-S), C; \quad E=T, X.$

We can observe that, for what concerns $\left(z_i - z_j\right)$ the contribute to axiom violation

i) is zero whenever the sign of $\left(t_i - t_j\right)$ is the same as the sign of $\left(x_i - x_j\right)$, independently from the sign of $\left(z_i - z_j\right)$;

ii) is positive whenever the sign of $\left(z_i - z_j\right)$ is the same as the sign of $\left(t_i - t_j\right)$, and opposite to the sign of $\left(x_i - x_j\right)$;

iii) is negative whenever the sign of $\left(z_i - z_j\right)$ is the same as the sign of $\left(x_i - x_j\right)$, and opposite to the sign of $\left(t_i - t_j\right)$.

For what concerns the KL measure of axiom 3 violation, it can be decomposed as

$$AV_3^T = \left(G_Y - C_{Y|X}\right) = AAV_3^V + AAV_3^D + AAV_3^C, \quad (9)$$

where

$$AAV_3^V = \frac{\mu_{X-V}}{\mu_Y}\left(r_{X-V|Y} - r_{X-V|X}\right)G_{X-V},$$

$$AAV_3^D = \frac{\mu_{V-S}}{\mu_Y}\left(r_{V-S|Y} - r_{V-S|X}\right)G_{V-S}$$

$$AAV_3^C = \frac{\mu_C}{\mu_Y}\left(r_{C|Y} - r_{C|X}\right)G_C,$$

Here similar considerations apply as those presented for AV_1^T.

V. Conclusion

The instruments presented in this paper can be applied to investigate how the combination of the marginal tax schedule, of deduction and tax credits fulfill the aim of equity, according to KL axioms. The paper links together the contributions of Pfäler [7]; Lambert [3] and Onrubia et Al. [5], with the

axiomatic approach presented by Kakwani and Lambert [2]. Differently from Pfäler, Lambert and Onrubia et Al, we do not consider explicitly deductions, as we think that their effect can be better evaluated by inspecting the distribution of the tax revenue without deductions, versus the one obtained after their application.

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