Comparative Analysis of Symmetric Method, Weighted Additive Method and Weighted Max-Min Method for Fuzzy Multi-Objective Supplier Selection in a Supply Chain

G. S. Mada

Abstract— **Supplier selection is one of the most important activities in a company's purchasing department. Supplier selection is a multi-criteria issue. The criteria in question are not all quantitative but also qualitative. In fact, the decision maker does not have definite information regarding the criteria and limits of the decision. In this case, the fuzzy set theory is one of the best tools for dealing with uncertainty. The fuzzy set theory is applied to supplier selection problems because of the lack of clarity and inaccuracy of information. There are three approach models that can be used to solve the fuzzy multi-objective suppliers selection problem such as symmetrical method, weighted additive method and weighting max-min method. In determining weights of the objective function that describe the relative importance of criteria, the analytic hierarchy process (AHP) method is used. These three methods can be applied to help purchasing managers strategize the company to minimize purchase costs, maximize quality and maximize service. The applications of these three models are illustrated in numerical examples.**

Keywords—**Symmetric Method, Weighted Additive Method, Weighted Max-Min Method, Supply Chain, Fuzzy Mulyi-Objective Linear Programming.**

I. Introduction

Supply chain is a network consisting of geographically dispersed different facilities, where raw materials, semi-finished materials and finished materials are manufactured, tested, modified and stored, and there is a delivery process that connects these facilities. Business entities involved in a supply chain include suppliers of raw materials, production units, additional suppliers, storage service providers, collectors, distributors, retailers and consumers. The purpose of making the supply chain is to obtain a network that is in line with the objectives to be achieved by decision makers. The description of the supply chain is generally presented in Fig. 1. Supply Chain Management is an approach used to make performance from an entity consisting of a number of suppliers, factories, warehouses and stores to be efficient, in such a way that the product can be produced and distributed in the right quantities, and at the right time, in order to minimize the total cost while still providing satisfactory service. The main purpose of supply chain management is to meet customer demand through the most efficient use of resources, including distribution capacity, inventories, and human resources [7]. According to [2], Supplier selection is a multi-criteria problem that includes both qualitative and quantitative factors. The relative importance of the criteria and sub-criteria is determined by the top management and purchasing managers based on the supply chain strategy.

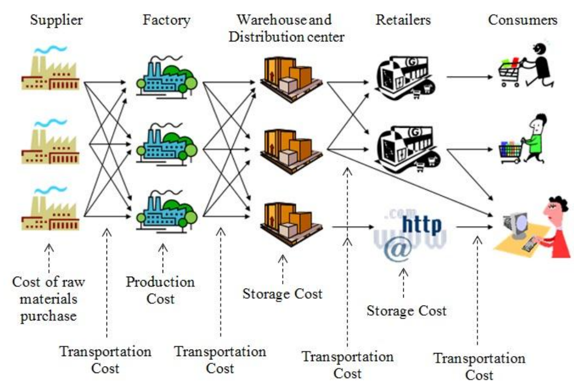

Fig. 1. Supply Chain

In fact, the decision maker does not have definite and complete information relating to the criteria and limits of the decision. In this case the fuzzy set theory is one of the best tools for dealing with uncertainty. The fuzzy set theory is applied to supplier selection problems because of the lack of clarity and inaccuracy of information. There are 3 models that can be used to complete the fuzzy multi-objective linear program, including the symmetric model of Zimmermann [1], the weighted additive model [1] and the max-min weighting model [2]. In symmetric model, each objective function is considered to have the same importance while in the weighted additive model and weighted max-min model, each objective function is considered to have different importance.

In this paper, the three models will be developed: symmetrical model, weighted additive model and weighting max-min model for fuzzy multi-objective suppliers selection to

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G. S. Mada adalah Candidate of Master of Mathematics at Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, Sekip Utara St., Sleman, Yogyakarta 55281, INDONESIA.

enable the purchasing managers to specify the number of orders to each supplier based on the supply chain strategy.

II. LITERATURE REVIEW

Supplier selection problem is a problem encountered in a supply chain. Simchi-Li et al. (2004), Pibernik and Sucky (2007) and Pokharel (2007) in [7] provide definitions of the supply chain and problem problems faced in a supply chain.

Reference [4] first identified and analyzed 23 important criteria for supplier selection based on survey of purchasing managers. Reference [11] reviews 74 articles that discuss the criteria of supplier selection. They also concluded that supplier selection is a multi-criteria problem and the priority of each criterion depends on each purchase situation. Based on 23 criteria declared by [4], [11] subsequently minimized the scope of the most important criteria of supplier selection based on its rating.

According to [2], supplier selection problem is a multi-objective programming problem. Reference [10] provides a definition of a multi-objective programing. In this paper only discussed about supplier selection problem with linear objective function and constraint function. In other words, the issue of supplier selection discussed is a multi-objective linear programming problem. Reference [8] provides definitions and methods for solving multi-objective linear programming problem. The supplier selection model expressed as a multi-objective linear programing is called a deterministic model.

In fact information relating to the objectives and constraints faced in the problem of supplier selection is not known with certainty. For this reason the deterministic model is no longer appropriate to represent the supplier selection problem. To overcome the uncertainty of objectives and constraints, [2] suggested to using fuzzy set theory. The fuzzy set was first introduced by Zadeh [8]. In [3] it is mentioned that Zimmermann uses the fuzzy set theory to express the uncertainty of the goals that the decision maker wants to achieve in the multi-objective linear programming problem. Multi-objective linear programming with a fuzzy objective function is called a fuzzy multi-objective linear programming.

According to [1], it was stated that Zimmermann had used the model proposed by Belman and Zadeh to solve the problem of a fuzzy multi-objective linear programming. In this model the objective function and constraint function is considered to have the same level of importance for the decision maker, so this model is called the symmetrical model. However, in general business activities, between one objective and the other objectives have different importance for decision makers, not least on supplier selection problem. Therefore, symmetrical model is no longer appropriate to solve the problem of decision making with some goals, because the goals have different level of important so that later [1] develop an weighted additive model for supplier selection problem with the aim of addressing improper inputs and the underlying problem of determining the weight of quantitative / qualitative criteria with the conditions of multiple sources and capacity constraints. In the weighted additive model, there is no guarantee that the level of achievement of fuzzy goals is consistent with the desired weight or expected decision maker. When the decision maker determines the weight of the objective function, the ratio of the achievement level of the membership function should be as close as possible to the objective weight ratio to reflect the relative importance of the criterion. However in the weighted additive model, the ratio of achievement levels is not necessarily the same as the weight of the objective function. Then [2] developed a weighted max-min model in order to overcome the problem of difference in achievement rate ratio and objective functional weights.

In this paper, the three models will be rebuilt and applied to the problem of fuzzy multi-objective suppliers selection. The Analytic Hierarchy Process (AHP) method is often used to solve complex problems and has been applied in various decision-making contexts [9]-[10]. AHP also provides a structured approach to determine the criteria weight. AHP is used to determine the weight of the criteria in the model presented.

III. MULTI-OBJECTIVE SUPPLIER SELECTION MODEL

Let n states the number of suppliers that can be chosen by the decision maker and $x_i, i = 1, 2, \cdots, n$ states the number of products to be purchased from the i-th supplier. In other words $x_i, i = 1, 2, \cdots, n$ is a decision variable. From here a multi-objective linear programming for supplier problems can be expressed as a problem of determining $\boldsymbol{x} = \left[x_1, x_2 \cdots, x_n\right]^T$ which meets

$$\text{Min } z_1(x), z_2(x), \cdots, z_p(x) \tag{1}$$

$$\text{Max } z_{p+1}(x), z_{p+2}(x), \cdots, z_q(x) \tag{2}$$

subject to:

$$x \in X_d,\ X_d = \left\{ x \mid g_r(x) = \sum_{i=1}^{n} a_{ri} x_i \le b_r,\ r = 1, 2, \cdots, h \right\} \tag{3}$$

where $z_1(x), z_2(x), \cdots, z_p(x)$ is the objective function will be minimized such as cost, delivery delay, and others, and $z_{p+1}(x), z_{p+2}(x), \cdots, z_q(x)$ is the goal to be maximized such as quality, delivery accuracy, service level, and others, $g_r(x)$ is a function of constraints that must be met such as demand from buyers, supplier capacity, and others, X_d is a visible area that meets the constraints.

It is clear that the supplier selection problem is an optimization problem that requires the formulation of the objective function. Not all criteria in this problem are quantitative. This problem is recognized by Ghodsypour and O'Brien (1998) in [2]. They propose an integrated method that uses Analytic Hierarchy Process to handle both qualitative and quantitative criteria. A comprehensive review of supplier selection criteria is presented by Ghodsypour and O'Brien

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(1996) in [2]. They conclude that the number and weighting criteria depend on the purchasing strategy.

To have a similar model, in this paper the purchasing criteria considered are quality, cost of purchase and service level. These criteria are cited most often in ordering decisions (Roa and Kiser (1980); Ghodsypour and O'Brien (1998) in [2]).

Given the problem of supplier selection, with the number of suppliers to choose from as many as *n*. Each supplier has as much capacity $C_i, i = 1, 2, \cdots, n$. Number of demand to be met by decision makers as much as *D*. Decision maker want to determine which suppliers to choose and how many products to order from each selected supplier. Objectives to be achieved by decision makers is to minimize the purchase price, maximize product quality and maximize the level of service to be obtained from suppliers. The supplier selection problem is illustrated in Fig. 2.

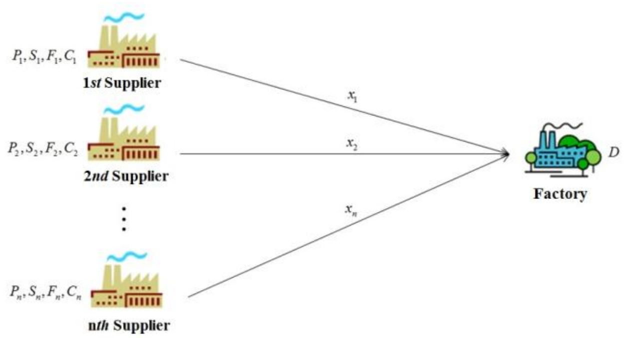

Fig. 2. Supplier Selection

Here's a mathematical model of the above problem.

Determine $\boldsymbol{x} = [x_1, x_2 \cdots, x_n]^T$ that meets:

$$\text{Min } z_1 = \sum_{i=1}^{n} P_i x_i \tag{4}$$

$$\text{Max } z_2 = \sum_{i=1}^{n} F_i x_i \tag{5}$$

$$\text{Max } z_3 = \sum_{i=1}^{n} S_i x_i \tag{6}$$

subject to:

$$\sum_{i=1}^{n} x_i \geq D \tag{7}$$

$$x_i \leq C_i, i = 1, 2, \cdots, n \tag{8}$$

$$x_i \geq 0, i = 1, 2, \cdots, n \tag{9}$$

The supplier selection problem has three objective functions: minimize purchase cost (4), maximize product quality (5) and maximize service obtained from supplier (6). Constraints (7) ensure demand is met. Constraint (8) states that the order amount of each supplier shall be equal to or less than the capacity of each supplier. Constraint (9) prohibits the number of reservations with negative value.

In fact, the goal to be achieved in the problem of supplier selection is not always certain but fuzzy. To overcome this problem, a supplier selection model with fuzzy objective function was developed.

An objective function is called an fuzzy objective function if the decision maker gives a fuzzy goal to the objective function. In the problem of supplier selection, the fuzzy goal given by the decision maker is "the value of objective function $z_i(x)$ substantially smaller or equal to an aspiration value given by the decision maker, z_i^0". This fuzzy goal is called fuzzy min. Another fuzzy goal is "the value of objective function $z_i(x)$ substantially greater than or equal to an aspiration value given by the decision maker, z_i^0". This fuzzy goal is called fuzzy max.

Furthermore the linear programming problem (1)-(3) with fuzzy goal can be expressed as a problem

Determine $\boldsymbol{x} = [x_1, x_2 \cdots, x_n]^T$ that meets

$$\text{Min } z_k = \sum_{i=1}^{n} c_{ki} x_i \tilde{\leq} z_k^0, \; k = 1, 2, \cdots, p \tag{10}$$

$$\text{Max } \tilde{z}_l = \sum_{i=1}^{n} c_{li} x_i \tilde{\geq} z_k^0, \; l = p+1, p+2, \cdots, q \tag{11}$$

subject to:

$$g_r(x) = \sum_{i=1}^{n} a_{ri} x_i \leq b_r, \; r = 1, 2, \cdots, h \tag{12}$$

$$x_i \geq 0, \; i = 1, 2, \cdots, n \tag{13}$$

where c_{ki}, c_{li}, a_{ri} and b_r are crisp, z_k^0 and z_l^0 are the level of aspiration the decision maker wants to achieve and tilde mark □ declared a fuzzy environment. The membership function for each fuzzy goal is a linear function as follows:

$$\mu_{zk}(x) = \begin{cases} 1 & ; \quad z_k \leq z_k^- \\ f_{\mu zk} = \dfrac{z_k^+ - z_k(x)}{z_k^+ - z_k^-} & ; \quad z_k^- \leq z_k(x) \leq z_k^+, (k = 1, 2, \cdots, p) \\ 0 & ; \quad z_k \geq z_k^+ \end{cases} \tag{14}$$

$$\mu_{zl}(x) = \begin{cases} 1 & ; \quad z_l \geq z_l^+ \\ f_{\mu zl} = \dfrac{z_l(x) - z_l^-}{z_l^+ - z_l^-} & ; \quad z_l^- \leq z_l(x) \leq z_l^+, (l = p+1, p+2, \cdots, q) \\ 0 & ; \quad z_l \leq z_l^- \end{cases} \tag{15}$$

where z_l^+, z_l^-, z_k^+ and z_k^- each representing the value of the objective function $z_l(x)$ dan $z_k(x)$, such that membership degrees 1 and 0. Membership function for z_k and z_l shown in Fig. 3.

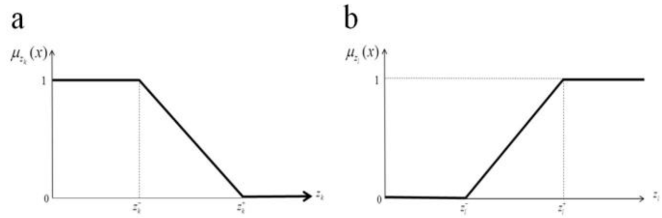

Fig. 3. Membership Function for (a) Min z_k and (b) Max z_l .

In completing the linear programming (10)-(13) not all objective functions simultaneously achieve optimum fuzzy with the constraints provided. So in practice decision makers usually choose Pareto optimal solution as a final decision based on the degree of satisfaction (or level of fuzzy aspiration) at each goal. Pareto optimal solution for linear programming (10)-(13) is defined as follows:

Definition 1. *Visible solution x^* on a linear programming (10)-(13) called pareto optimal solution if and only if there is no other visible solution x^o so for each $j,\left(j=1,2,\cdots,k\right)$ apply $\mu\left(z_j\left(x^o\right)\right)\geq\mu\left(z_j\left(x^*\right)\right)$ and $\mu\left(z_j\left(x^o\right)\right)\neq\mu\left(z_j\left(x^*\right)\right)$ for at least one $j,\left(j=1,2,\cdots,k\right)$.*

IV. Solving Fuzzy Multi-Objective Supplier Selection Problem

There are three models that can be used to solve the fuzzy multi-objective suppliers selection problem such as:

A. The Symmetric Model

This model was developed by Zimmerman [1]. In this model the objective function is considered to have the same importance. The model formulation is as follows:

Determine $\boldsymbol{x}=\left[x_1,x_2\cdots,x_n\right]^T$ that meets

Max λ

subject to:

$$\begin{aligned}
&\lambda\leq f_{\mu zj}\left(x\right),\ j=1,2,\cdots,q\\
&g_r\left(x\right)\leq b_r,\ r=1,2,\cdots,h\\
&x_i\geq 0,\ i=1,2,\cdots,n\\
&0\leq\lambda\leq 1
\end{aligned}\tag{16}$$

B. The Weighted Addtive Model

This model was developed by [2]. This model is a nonsimetris model because for each objective function is considered to have different importance. The model formulation is as follows:

Determine $\boldsymbol{x}=\left[x_1,x_2\cdots,x_n\right]^T$ that meets

Max $\sum_{j=1}^{q} w_j\lambda_j$

subject to:

$$\begin{aligned}
&\lambda_j\leq f_{\mu zj}\left(x\right),\ j=1,2,\cdots,q\\
&g_r\left(x\right)\leq b_r,\ r=1,2,\cdots,h\\
&\lambda_j\in\left[0,1\right],\ j=1,2,\cdots,q\\
&\sum_{j=1}^{q} w_j=1, w_j\geq 0\\
&x_i\geq 0,\ i=1,2,\cdots,n
\end{aligned}\tag{17}$$

where *w* is the weight of the objective function.

C. The Weighted Max-Min Model

This model was developed by [2] to improve the weighted additive model that does not guarantee consistency between the achievement level of the objective function and the functional weight. The model formulation is as follows:

Determine $\boldsymbol{x}=\left[x_1,x_2\cdots,x_n\right]^T$ that meets

Max λ

subject to:

$$\begin{aligned}
&w_j\lambda\leq f_{\mu zj}\left(x\right),\ j=1,2,\cdots,q\\
&g_r\left(x\right)\leq b_r,\ r=1,2,\cdots,h\\
&\sum_{j=1}^{q} w_j=1, w_j\geq 0\\
&x_i,\lambda\geq 0,\ i=1,2,\cdots,n
\end{aligned}\tag{18}$$

where *w* is the weight of the objective function.

Linear programming (16), (17) and (18) are deterministic linear programming or ordinary linear programming, so they can be solved by the simplex method. For more details about the transformation of the fuzzy multi-objecttive linear programming into a deterministic single-objective linear program can be seen in [1]-[2]. By completing (16), (17) and (18) obtained the optimal solution λ^* and x^*. The optimal solution x^* x^* is the optimal solution of linear programming (10)-(13) and λ^* is the membership degree for $z_j\left(x^*\right),\ j=1,2,\cdots,q$. The following two theorems will be shown to ensure that the optimal solution obtained from the weighted max-min model (18) is the optimal solution of the linear programming (10)-(13) and with the same analogy can be developed for the other two models.

Theorem 1. *If $x^*\in X$ is the unique optimal solution of weighted max-min model (18) for some $w=\left(w_1,\cdots,w_q\right)>0$, then x^* is the Pareto optimal solution of the linear programming (10)-(13).*

Proof. Is known that x^* is unique optimal solution of weighted max-min problem (18) for some $w=\left(w_1,\cdots,w_q\right)>0$, based on (18) it means x^* the only member of X that meets

$x^* = \max \min_{j=1,2,\cdots,q} \frac{f_{\mu zj}(x)}{w_j}$. If x^* is not Pareto optimal solution of the linear programming (10)-(13), then there exists $x^o \in X$ such that $f_{\mu zi}(x^o) < f_{\mu zi}(x^*)$ for some i and $f_{\mu zj}(x^o) \le f_{\mu zj}(x^*)$, $j = 1, 2, \cdots q; j \ne i$.

So $\frac{f_{\mu zi}(x^o)}{w_j} < \frac{f_{\mu zi}(x^*)}{w_j}$ for some i and

$\frac{f_{\mu zi}(x^o)}{w_j} \le \frac{f_{\mu zi}(x^*)}{w_j}$, $j = 1, 2, \cdots q; j \ne i$. If the minimum value is taken for $j = 1, 2, \cdots q$ obtained:

$$\min\left\{\frac{f_{\mu zi}(x^o)}{w_j}\right\} \le \min\left\{\frac{f_{\mu zi}(x^*)}{w_j}\right\}.$$

Furthermore, if the maximum value is taken, obtained:

$$\max\left\{\min\left\{\frac{f_{\mu zi}(x^o)}{w_j}\right\}\right\} \le \max\left\{\min\left\{\frac{f_{\mu zi}(x^*)}{w_j}\right\}\right\}.$$

This contradicts the statement x^* is the single optimal solution of the weighted max-min model (18). Hence, x^* is the Pareto optimal solution of the linear programming (10)-(13).

■

Theorem 2. *If $x^* \in X$ is the Pareto optimal solution of the linear programming (10)-(13), then x^* is the optimal solution of the weighted max-min model (18) for some* $w = (w_1, \cdots, w_q) > 0$.

Proof. Is known that $x^* \in X$ is Pareto optimal solution of the linear programming (10)-(13), then by Definition 1, it,s means that $f_{\mu zj}(x^*) \le f_{\mu zj}(x^o), x^o \in X$. Next, we choose a weight $w^* = (w_1^*, \cdots, w_q^*) > 0$ such that $\frac{f_{\mu zi}(x^*)}{w_j^*} = \lambda,\ j = 1, 2, \cdots q$.

For this weight, if x^* is not the optimal solution of the weighted max-min model (18), then there exists $x^o \in X$ so

$$\max\left\{\min\left\{\frac{f_{\mu zi}(x^o)}{w_j}\right\}\right\} \le \max\left\{\min\left\{\frac{f_{\mu zi}(x^*)}{w_j}\right\}\right\}.$$

As a consequence, there exists $x^o \in X$ so $f_{\mu zj}(x^o) \le f_{\mu zj}(x^*)$, $j = 1, 2, \cdots q$. This is a contradiction with x^* x^* as Pareto optimal solution of linear programming (10)-(13). Hence, x^* is the optimal solution of the weighted max-min model (18).

■

The value of the new membership function and the optimal level of achievement (λ^*) can exceed the current unity when $w_j < 1$. However, the actual level of achievement for each objective function may never exceed unity. This model produces the optimal solution in the visible region so that the ratio of achievement levels of the membership function is as close as possible to the ratio of objective function weights [6].

For getting the weight or importance level between each fuzzy goal of the decision maker is a very important initial process to complete this model. For determine weights of the objective function in this paper are used Analytic Hierarchy Process (AHP).

V. Analytic Hierarchy Process (AHP) and Algorithm for Solving Fuzzy Multi-Objective Supplier Selection Problem

A. Analytic Hierarchy Process (AHP)

Algorithm for determine weights of objective functions are as follows:

1. Create a pairwise comparison matrix of the importance of each objective function desired by the decision maker.
2. If the comparison matrix of a consistently perfect decision maker then the next step is the determine vector $\boldsymbol{w} = [w_1\ w_2\ \dots\ w_n]$ that is nontrivial solution of the system of n equations $A\boldsymbol{w}^T = \Delta \boldsymbol{w}^T$ with Δ is an unknown number and is an unknown n-dimension column matrix.
3. If the matrix of comparison from the decision maker is not consistent then the matrix $\boldsymbol{w}$ estimated based $\boldsymbol{w}_{maks}$.
4. Conduct consistency test on comparison matrices.

For more details about algorithm to determine weights of objective function with AHP, it can be seen in [10]-[9].

B. Algorithm to solving the fuzzy multi-objective supplier selection.

***Step* 1**: Declare the problem of supplier selection as a multi-objective linear programming based on the goals to be achieved and the constraints faced.

***Step* 2**: Determine minimum and maximum individual for each objective function based on the existing constraints.

***Step* 3**: Ask the decision maker about the goal and the value of the constraints that want to be achieved along with the value of tolerance and leniency for the goal. Goal with such leeway is called fuzzy goal.

***Step* 4**: Determine membership function for fuzzy objective function based on fuzzy goal given by decision maker.

***Step* 5**: If all the objective functions have the same importance then the fuzzy multi-objective supplier selection problem is directly transformed into a deterministic single-objective linear programming using the equation (16) and proceed directly to Step 8.

***Step* 6**: If each objective function has a different level of importance then asks the decision maker to determine the weight of each fuzzy goal by using the Analytic Hierarchy Process approach.

***Step* 7**: Transform the supplier selection problem of a fuzzy multi-objective linear program becomes a single-objective deterministic linear programming problem as in equation (17) or equation (18).

***Step* 8**: Determine optimal solution x^* by solving the deterministic single-objective linear programming problem using Simplex method.

This algorithm is illustrated in the following numerical example.

VI. Numerical Example

For supplying a new product to the market, it is assumed that there are three suppliers to choose from by a decision maker. All three suppliers have different capabilities and capacities in providing the new product. Criteria for purchasing products from these three suppliers are the cost of purchase, quality and service.

Goal to be achieved by the decision makers of the purchase of new products is to minimize the cost of purchase, maximize the quality of products obtained and maximize the level of service that will be obtained from suppliers. Supplier capacity constraints are also considered in the model. It is assumed that the information on supplier performance on the above criteria is not accurately known. The estimated value of cost, quality and service levels and supplier constraints are presented in Table 1. Market demand to be met is 1000 tons.

The model formulation of supplier selection problem above is as follows:

Min $Z_1 = 13x_1 + 11.5x_2 + 15x_3$

Max $Z_2 = 0.8x_1 + 0.7x_2 + 0.9x_3$

Max $Z_3 = 0.85x_1 + 0.75x_2 + 0.8x_3$

subject to:

$x_1 + x_2 + x_3 = 1000$

$x_1 \le 700, \quad x_2 \le 600, \quad x_3 \le 500$

$x_i \ge 0, i = 1, 2, 3$

Three objective functions Z_1, Z_2 and Z_3 cost, quality and service, respectively, and x_i is the number of units purchased from the *i*th supplier.

Table II presents the individual maximums and minimums of the three objective functions, the maximum and minimum individual values then became fuzzy goals for each objective function. Equations (19), (20) and (21) show the membership function of the three fuzzy goals.

TABLE I: Suppliers' Quantitative Information

	Price (million rupiah/ton)	Quality (%)	Service (%)	Capacity (ton)
Supplier 1	13	80	85	700
Supplier 2	11.5	70	75	600
Supplier 3	15	95	80	500

Furthermore, the problem of supplier selection is solved by three different methods, weightless method (symmetric), weighted additive method and weighted max-min method whose algorithm has been formed in the previous section.

TABLE II: The Data Set for Membership Functions

Objective Function	$\mu = 0$	$\mu = 1$	$\mu = 0$
Z_1 (Purchase Cost)	-	12100	14000
Z_2 (Quality Level)	740	875	-
Z_3 (Service Level)	770	835	-

For $Z_1 = 13x_1 + 11.5x_2 + 15x_3$ (Purchase Cost) :

$$\mu_{z1}(x) = \begin{cases} 1 & ; \quad z_1 \le 12100 \\ \dfrac{14000 - z_1}{1900} & ; \quad 12000 \le z_1 \le 14000 \\ 0 & ; \quad z_1 \ge 14000 \end{cases} \tag{19}$$

For $Z_2 = 0.8x_1 + 0.7x_2 + 0.9x_3$ (Quality Level)

$$\mu_{z2}(x) = \begin{cases} 1 & ; \quad z_2 \ge 875 \\ \dfrac{z_2 - 740}{135} & ; \quad 740 \le z_2 \le 875 \\ 0 & ; \quad z_2 \le 740 \end{cases} \tag{20}$$

For $Z_3 = 0.85x_1 + 0.75x_2 + 0.8x_3$ (Service Level):

$$\mu_{z3}(x) = \begin{cases} 1 & ; \quad z_3 \ge 835 \\ \dfrac{z_3 - 770}{65} & ; \quad 770 \le z_3 \le 835 \\ 0 & ; \quad z_3 \le 770 \end{cases} \tag{21}$$

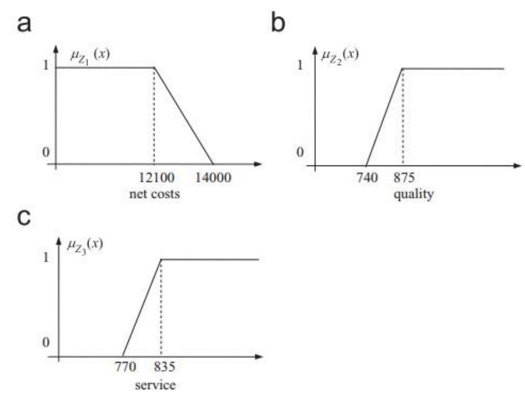

Fig. 4. Membership Function (a) Purchase Cost, (b) Quality Level, and (c) Service Level.

For Symmetrical Model:

Deteremine $[x_1, x_2, x_3]^T$ that meets

Max λ

subject to:

$$\lambda \le \frac{14.000 - (13x_1 + 11.5x_2 + 15x_3)}{1900}$$

$$\lambda \le \frac{(0.8x_1 + 0.7x_2 + 0.9x_3) - 770}{135}$$

$$\lambda \le \frac{(0.85x_1 + 0.75x_2 + 0.8x_3) - 740}{65}$$

$$x_1 + x_2 + x_3 = 1000$$

$$x_1 \le 700, \quad x_2 \le 600, \quad x_3 \le 500$$

$$x_i \ge 0, i = 1, 2, 3$$

$$\lambda \in [0,1]$$

For Weighted Additive Model:

Determine $[x_1, x_2, x_3]^T$ that meets

Max $0.63\lambda_1 + 0.11\lambda_2 + 0.26\lambda_3$

subject to:

$$\lambda_1 \le \frac{14.000 - (13x_1 + 11.5x_2 + 15x_3)}{1900}$$

$$\lambda_2 \le \frac{(0.8x_1 + 0.7x_2 + 0.9x_3) - 770}{135}$$

$$\lambda_3 \le \frac{(0.85x_1 + 0.75x_2 + 0.8x_3) - 740}{65}$$

$$x_1 + x_2 + x_3 = 1000$$

$$x_1 \le 700, \quad x_2 \le 600, \quad x_3 \le 500$$

$$x_i \ge 0, i = 1, 2, 3$$

$$\lambda_j \in [0,1], \; j = 1, 2, 3$$

For Weighted Max-Min Model:

Determine $[x_1, x_2, x_3]^T$ that meets

Max λ

subject to:

$$0.63\lambda \le \frac{14.000 - (13x_1 + 11.5x_2 + 15x_3)}{1900}$$

$$0.11\lambda \le \frac{(0.8x_1 + 0.7x_2 + 0.9x_3) - 770}{135}$$

$$0.26\lambda \le \frac{(0.85x_1 + 0.75x_2 + 0.8x_3) - 740}{65}$$

$$x_1 + x_2 + x_3 = 1000$$

$$x_1 \le 700, \quad x_2 \le 600, \quad x_3 \le 500$$

$$x_1, x_2, x_3, \lambda \ge 0$$

With the help of POM for Windows software, obtained the results as presented in Table III.

Based on Table III, on a weightless approach or a symmetrical approach, there is no difference of importance between the three criteria so the objective function having the same weight, resulting in the level of attainment for all objective functions is same, $\mu_{Z1} = \mu_{Z2} = \mu_{Z3} = 0.9$. Furthermore, Table III shows that the weighted additive model is unacceptable because the achievement levels are not corresponding to the weight of the objective function. The achieved level of the first objective function is lower than the achieved level of the second objective function even though the weight of first objective function is greater than the weight of the second objective function. Comparing the solution obtained by the weighted max-min approach, it can be seen that the proposed model succeeded in finding the optimal solution such that the ratio of the achievement level of the objective function is equal to the ratio of the weight of its objective function and the solution more consistent than the other approach solution with the decision maker's preference or expectation. In other word $(\mu_1 > \mu_3 > \mu_2)$ agrees with $(w_1 > w_3 > w_2)$.

TABLE III: THE CALCULATIONS RESULTS OF NUMERICAL EXAMPLE WITH THREE DIFFERENT APPROACHES

	Method 1	Method 2	Method 3
Z_1	12380	13600	13836
Z_2	760	845	863
Z_3	793	835	829
x_1	386	700	582
x_2	528	0	0
x_3	86	300	418
μ_1	0.85	0.21	0.9
μ_2	0.15	0.77	0.9
μ_3	0.35	1	0.9

where Method 1 is The Weighted Max-Min Method, Method 2 is The Weighted Additive Method and Method 3 is The Weightless Method (Symmetric Method).

VII. Conclusion

Based on the application of the three methods in the numerical example it can be concluded that the weighted max-min method is the best method to solving the fuzzy multi-objective supplier selection problem.

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G.S.Mada was born on Rote Island, Indonesia on February 24, 1993. He completed his undergraduate education at the Department of Mathematics, Faculty of Science and Engineering, Nusa Cendana University, Kupang Indonesia in June 2016 and then continued his Master's study at the Mathematics Department of the Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, Yogyakarta Indonesia by concentrating on the interest of Applied Mathematics.

He worked as TEACHING ASSISTANT at Nusa Cendana University, Kupang Indonesia throughout 2015 for several courses such as Dynamic Systems, Mathematical Statistics for Biology, and Mathematical Research Methods. Then he became a Lecturer staff at the University of Nusa Lontar, Rote who oversaw several lectures such as Calculus for General and Engineering, Mathematical Statistics, and Computer Introduction until April 2016. He won the second place of the Pertamina’s MIPA Olympic Selection through his paper, "Differential Equations Model to investigate whether Pertamina Abherka's Floating Storage and Offloading (FSO) construction can guarantee the smooth operation of Pertamina Upstream Oil and Gas Production of West Madura Offshore (PHE WMO) ". His previous research is "Ananlisis Comparison Fuzzy Inerence System Tsukamoto and Fuzzy Inerence System Mamdani in Determine Production Quantity", which is his thesis at Nusa Cendana University, Kupang Indonesia. He is interested in the fields of Operations Research, Optimization Theory and Fuzzy Multi-Objective Linear Programming.

Mr. Mada is now temporarily continuing her Master's education with financing from the Government of Indonesia through Lembaga Pengelolah Dana Pendidikan (LPDP).