# Analyzing Free Vibration of Tapered Transversally Functionally Graded Beams Using a Stepped Approach

Ziou H. and Guenfoud M.

***Abstract*—This paper explores the vibration characteristics of functionally graded tapered beams. A straightforward stepped approach is proposed to examine the behavior of these beams. The method involves dividing the non-uniform beams into segments with uniform cross-sections for analysis. A finite element model based on the Euler–Bernoulli theory is developed to assess the structural behavior of slender beams. Functionally graded tapered beams are defined by smooth material property variations as a result of continuous changes in material properties, achieved through continuous changes in mechanical properties that follow a power law distribution across the beam's thickness. The finite element equations are derived using the principle of virtual work. The shape functions and stiffness matrix of the beam are detailed for the first time. Numerical results from the finite element analysis are presented in both tables and graphs to highlight the impact of the power law index, tapering parameter, Young's modulus variation, beam length, and boundary conditions on the vibration behavior of these beams. Additionally, variations of mode shapes are provided to qualitatively assess the slope and the deflection.**

***Keywords*—Functionally Graded Material, Mode shapes Tapered beam, Stepped approach.**

## I. Introduction

Structural elements with different cross-sections are widely used in civil engineering, mechanical engineering and aerospace as they can improve strength and structural stability and sometimes meet architectural requirements for aesthetics, functionality, weight reduction and cost efficiency. A major advancement in this field is the use of beams with variable cross-sections made of different materials such as steel, wood and composites. Moreover, the introduction of functionally graded materials (FGMs) has expanded the possibilities in this field, providing more opportunities for innovation and efficiency. Many researchers have devoted considerable efforts to deeply investigate the static and dynamic properties of functionally graded beams with different cross-sectional profiles. Li and Li [1] developed the Timoshenko-Euler element using equilibrium differential equations and also considered the effect of shear deformation when studying the stability of linearly variable cross-section beams under axial compression loads. Li [2] studied the static and dynamic characteristics of functionally graded beams using a unified approach through analytical methods. Mohanty et al. [3] evaluated the static and dynamic performance of P-FGM beams and sandwich beams using the finite element method based on the first-order shear deformation theory. Huang and Li [4] proposed a new approach based on transforming the differential equation of motion for transverse vibration of non-uniform Euler-Bernoulli beams into a Fredholm integral equation to study the free vibration of tapered beams under different conditions. Bayat et al. [5] used the Maximum-Minimum Method (MMA) and the homotopy perturbation method (HPM) to analyze the nonlinear free vibration (large amplitude) of a tapered beam and obtained the natural frequency and corresponding displacement of the tapered beam.

The aim of this study is to pioneer a straightforward incremental technique based on the finite element method specifically designed for the dynamic analysis of transversally functionally graded (TFG) I-beams, using Euler-Bernoulli theory. Tabular and graphical results are provided to underscore the effects of different parameters, including variations in material properties, tapering ratios, and end boundary conditions, on the dynamic behavior of these beams. The results presented in this paper will be a valuable point of reference for future studies investigating the dynamic behavior of functionally graded tapered I-beams.

## II. Mathematical formulation

### A. Functionally graded materials:

The dimensions of the analyzed FG beam are represented by L, b, and h in the x, y, and z directions, respectively. The effective material properties of the beam such as Young's modulus and mass density are represented with an additional term to incorporate porosity in the thickness direction as follows:

$$E(z) = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^k - \phi(z) \tag{1}$$

Here, E represent Young's modulus. The subscripts c and m refer to the ceramics and metal, respectively, while k signifies the power-law index or volume fraction index. The material assumes isotropic properties when k equals 0.

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*B. Classical beam theory:*

*Displacement field:*

Based on Euler-Bernoulli beam theory, the displacement field is written as:

$$u(x,z,t) = u_0(x,t) - z\theta(x,t)$$
$$w(x,z,t) = w_0(x,t) \tag{2}$$

*Strain and stress fields:*

The strain field for a 3D solid is determined as follows:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} = \varepsilon_{xx}^0 - z\kappa_{xx}^0 \tag{3}$$

$\varepsilon_{xx}^0$ is the extensional strain and $\kappa_{xx}^0$ is the bending strain.

The beam is under a pure axial strain state $\varepsilon_{xx}$. The axial stress $\sigma_{xx}$ is related to $\varepsilon_{xx}$ by Hook's Law as:

$$\sigma_{xx} = E(z)\varepsilon_{xx} = E(z)\left[\frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2}\right] = E(z)\left[\varepsilon_{xx}^0 - z\kappa_{xx}^0\right] \tag{4}$$

*Equation of motion:*

Hamilton's principle states that:

$$\int_{t_1}^{t_2}(\delta U - \delta T)\,dt = 0 \tag{5}$$

Where δU and δT are the variations of the virtual strain energy and the virtual kinetic energy, respectively.

## III. STEPPED APPROACH TO TAPERED BEAM DIVISION

The stepped approach approximates the tapered beam as a series of uniform segments. This allows established methods for analyzing uniform beams, such as shape functions and the stiffness matrix, to be applied to each segment. Consider the tapered beam, which is subdivided into uniform segments as illustrated in Fig. 1.

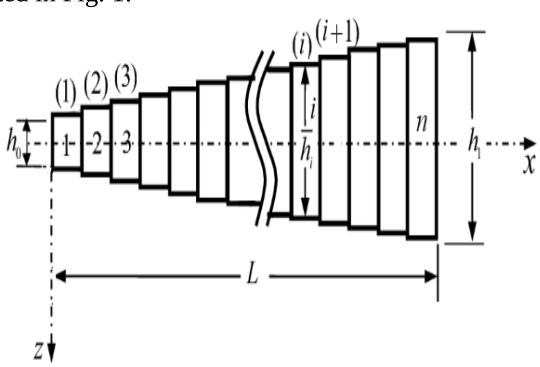


Fig. 1. Subdivision of the tapered beam into uniform segments.

The cross-section and moment of inertia at each node can be determined by using linear equations and incorporating data from the boundary conditions.

$$A(x) = ax + b \tag{6}$$

$$h(x) = ax + b = h_0\left(1 + \alpha\frac{x}{L}\right) \tag{7}$$

Where $\alpha$ is the tapering parameter

$$\alpha = \frac{h_0 - h_1}{h_1}$$

The moment of inertia can be expressed in terms of $h(x)$ as:

$$h(x) = ax + b = h_0\left(1 + \alpha\frac{x}{L}\right);\ I(x) = \frac{bh_0^3}{12}\left(1 + \alpha\frac{x}{L}\right)^3 \tag{8}$$

Basically, each segment has two nodes, and each of these nodes have their locations so the cross-section area and the moment of inertia of the uniform beam segment are determined from the average cross-section and the average moment of inertia of the corresponding nodes of beam segment, respectively.

$$A(element\ i) = \frac{(A_i + A_{i+1})}{2};\ I(element\ i) = \frac{(I_i + I_{i+1})}{2} \tag{9}$$

## IV. NUMERICAL RESULTS AND DISCUSSIONS

*Example 1:*

In the first example, a doubly tapered symmetric simply supported beam subjected to free vibration is investigated. The beam has a linear taper, that goes from $h_0$ to $h_1$, with a constant width (b=0.2). The height tapers with a ratio α, taking on the values 0.5 and 0.66. A value of α = 0.5 corresponds to a beam with a linearly varying depth from 0.3 to 0.6 meters (the depth is doubled). The second value α = 0.66 corresponds to a beam with a width of 0.2 meters and a varying depth from 0.3 to 0.9 meters (the depth is tripled). The dimensional and geometrical properties of the beam are shown in Fig. 2.

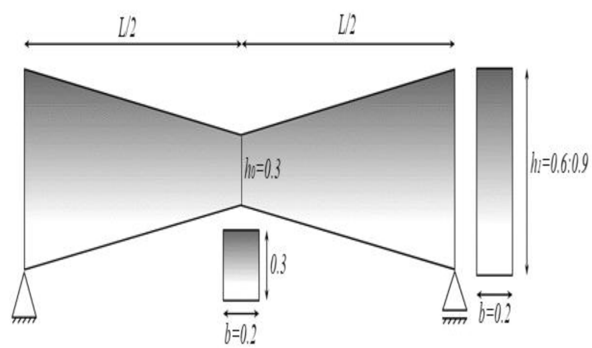


Fig. 2. Doubly tapered symmetric simply supported beam

*A. The effect of Young's modulus variation:*

The effects of Young's modulus ratios, beam length, non-negative power-law index, and tapering parameter on the first three frequencies of a simply-supported tapered FGM beam are presented in Tables 1–3. It is observed that natural frequencies increase with an increase in the power-law index when the $E_{ratio}$ is less than 1 ($E_{ratio} < 1$), and decrease with an increase in the power-law index when $E_{ratio}$ is greater than 1 ($E_{ratio} > 1$). For a constant power-law index, an increase in $E_{ratio}$ results in a decrease in fundamental frequencies. There is no significant variation in frequencies for an isotropic beam or when $E_{ratio}$ equals 1. As the length of the tapered beam increases, the fundamental frequencies decrease. Additionally, for a constant length and $E_{ratio}$, an increase in the tapering parameter leads to an increase in the 1st frequency. For the 2nd frequency, and for short beams (L = 2) when $E_{ratio} < 1$, the tapering ratio has no significant effect. However, the tapering ratio has a more significant effect on the 3rd frequency compared to the 1st and 2nd frequencies.

TABLE I: THE FIRST FREQUENCY PARAMETERS $\lambda_1$ OF S-S TAPERED BEAM FOR DIFFERENT MATERIAL DISTRIBUTION ($E_{ratio} = E_u/E_l$, $\rho_{ratio} = \rho_u/\rho_l = 1$).

| L | $E_{ratio}$ | α | k=0 | k=1 | k=2 | k=3 | k=4 | k=5 |
|---|---|---|---|---|---|---|---|---|
| 2 | 0.25 | 0.5 | 7.0240 | 10.9583 | 11.7411 | 12.1535 | 12.4263 | 12.6256 |
| | | 0.66 | 8.1802 | 12.7187 | 13.6476 | 14.1419 | 14.4686 | 14.7064 |
| | 1.0 | 0.5 | 7.0240 | 7.0240 | 7.0240 | 7.0240 | 7.0240 | 7.0240 |
| | | 0.66 | 8.1802 | 8.1802 | 8.1802 | 8.1802 | 8.1802 | 8.1802 |
| | 4.0 | 0.5 | 7.0240 | 4.9160 | 4.3954 | 4.2269 | 4.1545 | 4.1133 |
| | | 0.66 | 8.1802 | 5.7405 | 5.1234 | 4.9170 | 4.8257 | 4.7734 |
| 6 | 0.25 | 0.5 | 0.8374 | 1.2792 | 1.3723 | 1.4233 | 1.4578 | 1.4835 |
| | | 0.66 | 1.0233 | 1.5424 | 1.6582 | 1.7232 | 1.7677 | 1.8008 |
| | 1.0 | 0.5 | 0.8374 | 0.8374 | 0.8374 | 0.8374 | 0.8374 | 0.8374 |
| | | 0.66 | 1.0233 | 1.0233 | 1.0233 | 1.0233 | 1.0233 | 1.0233 |
| | 4.0 | 0.5 | 0.8374 | 0.5974 | 0.5387 | 0.5202 | 0.5121 | 0.5072 |
| | | 0.66 | 1.0233 | 0.7386 | 0.6674 | 0.6442 | 0.6337 | 0.6271 |
| 10 | 0.25 | 0.5 | 0.3034 | 0.4626 | 0.4963 | 0.5148 | 0.5274 | 0.5367 |
| | | 0.66 | 0.3723 | 0.5597 | 0.6018 | 0.6255 | 0.6418 | 0.6540 |
| | 1.0 | 0.5 | 0.3034 | 0.3034 | 0.3034 | 0.3034 | 0.3034 | 0.3034 |
| | | 0.66 | 0.3723 | 0.3723 | 0.3723 | 0.3723 | 0.3723 | 0.3723 |
| | 4.0 | 0.5 | 0.3034 | 0.2168 | 0.1957 | 0.1890 | 0.1861 | 0.1843 |
| | | 0.66 | 0.3723 | 0.2695 | 0.2438 | 0.2355 | 0.2317 | 0.2292 |

TABLE II: THE SECOND FREQUENCY PARAMETERS $\lambda_2$ OF S-S TAPERED BEAM FOR DIFFERENT MATERIAL DISTRIBUTION ($E_{ratio} = E_u/E_l$, $\rho_{ratio} = \rho_u/\rho_l = 1$).

| L | $E_{ratio}$ | α | k=0 | k=1 | k=2 | k=3 | k=4 | k=5 |
|---|---|---|---|---|---|---|---|---|
| 2 | 0.25 | 0.5 | 20.0929 | 29.6833 | 32.7155 | 34.3314 | 35.3516 | 36.0598 |
| | | 0.66 | 20.3606 | 29.6554 | 32.7607 | 34.4449 | 35.5208 | 36.2733 |
| | 1 | 0.5 | 20.0929 | 20.0929 | 20.0929 | 20.0929 | 20.0929 | 20.0929 |
| | | 0.66 | 20.3606 | 20.3606 | 20.3606 | 20.3606 | 20.3606 | 20.3606 |
| | 4 | 0.5 | 20.0929 | 15.7236 | 14.1421 | 13.4329 | 13.0122 | 12.7134 |
| | | 0.66 | 20.3606 | 16.4798 | 14.9150 | 14.1357 | 13.6424 | 13.2818 |
| 6 | 0.25 | 0.5 | 3.9012 | 6.4817 | 6.9119 | 7.0944 | 7.2016 | 7.2756 |
| | | 0.66 | 4.6479 | 7.4380 | 8.0256 | 8.3012 | 8.4671 | 8.5806 |
| | 1 | 0.5 | 3.9012 | 3.9012 | 3.9012 | 3.9012 | 3.9012 | 3.9012 |
| | | 0.66 | 4.6479 | 4.6479 | 4.6479 | 4.6479 | 4.6479 | 4.6479 |
| | 4 | 0.5 | 3.9012 | 2.6362 | 2.3453 | 2.2567 | 2.2212 | 2.2023 |
| | | 0.66 | 4.6479 | 3.2306 | 2.8730 | 2.7563 | 2.7046 | 2.6743 |
| 10 | 0.25 | 0.5 | 1.5071 | 2.5556 | 2.7022 | 2.7608 | 2.7954 | 2.8198 |
| | | 0.66 | 1.9003 | 3.1526 | 3.3568 | 3.4455 | 3.4984 | 3.5353 |
| | 1 | 0.5 | 1.5071 | 1.5071 | 1.5071 | 1.5071 | 1.5071 | 1.5071 |
| | | 0.66 | 1.9003 | 1.9003 | 1.9003 | 1.9003 | 1.9003 | 1.9003 |
| | 4 | 0.5 | 1.5071 | 0.9975 | 0.8872 | 0.8560 | 0.8449 | 0.8398 |
| | | 0.66 | 1.9003 | 1.2679 | 1.1261 | 1.0852 | 1.0702 | 1.0630 |

TABLE III: THE THIRD FREQUENCY PARAMETERS $\lambda_3$ OF S-S TAPERED BEAM FOR DIFFERENT MATERIAL DISTRIBUTION ($E_{ratio} = E_u/E_l$, $\rho_{ratio} = \rho_u/\rho_l = 1$).

| L | $E_{ratio}$ | α | k=0 | k=1 | k=2 | k=3 | k=4 | k=5 |
|---|---|---|---|---|---|---|---|---|
| 2 | 0.25 | 0.5 | 36.4224 | 59.6711 | 63.9221 | 65.8505 | 67.0131 | 67.8150 |
| | | 0.66 | 40.3090 | 67.5588 | 72.3089 | 74.3203 | 75.4648 | 76.2227 |
| | 1 | 0.5 | 36.4224 | 36.4224 | 36.4224 | 36.4224 | 36.4224 | 36.4224 |
| | | 0.66 | 40.3090 | 40.3090 | 40.3090 | 40.3090 | 40.3090 | 40.3090 |
| | 4 | 0.5 | 36.4224 | 26.2070 | 23.4012 | 22.1994 | 21.5399 | 21.1186 |
| | | 0.66 | 40.3090 | 28.0181 | 24.8095 | 23.5456 | 22.9036 | 22.5189 |
| 6 | 0.25 | 0.5 | 6.8459 | 10.0480 | 10.9327 | 11.4361 | 11.7754 | 12.0232 |
| | | 0.66 | 7.8074 | 11.9619 | 12.9099 | 13.4139 | 13.7464 | 13.9875 |
| | 1 | 0.5 | 6.8459 | 6.8459 | 6.8459 | 6.8459 | 6.8459 | 6.8459 |
| | | 0.66 | 7.8074 | 7.8074 | 7.8074 | 7.8074 | 7.8074 | 7.8074 |
| | 4 | 0.5 | 6.8459 | 5.3798 | 4.9003 | 4.6715 | 4.5286 | 4.4250 |
| | | 0.66 | 7.8074 | 6.0090 | 5.4503 | 5.1820 | 5.0175 | 4.9017 |
| 10 | 0.25 | 0.5 | 2.9248 | 4.2309 | 4.6025 | 4.8192 | 4.9675 | 5.0771 |
| | | 0.66 | 3.4357 | 5.0611 | 5.4913 | 5.7350 | 5.9004 | 6.0222 |
| | 1 | 0.5 | 2.9248 | 2.9248 | 2.9248 | 2.9248 | 2.9248 | 2.9248 |
| | | 0.66 | 3.4357 | 3.4357 | 3.4357 | 3.4357 | 3.4357 | 3.4357 |
| | 4 | 0.5 | 2.9248 | 2.2841 | 2.0901 | 2.0039 | 1.9509 | 1.9118 |
| | | 0.66 | 3.4357 | 2.6975 | 2.4667 | 2.3565 | 2.2868 | 2.2355 |

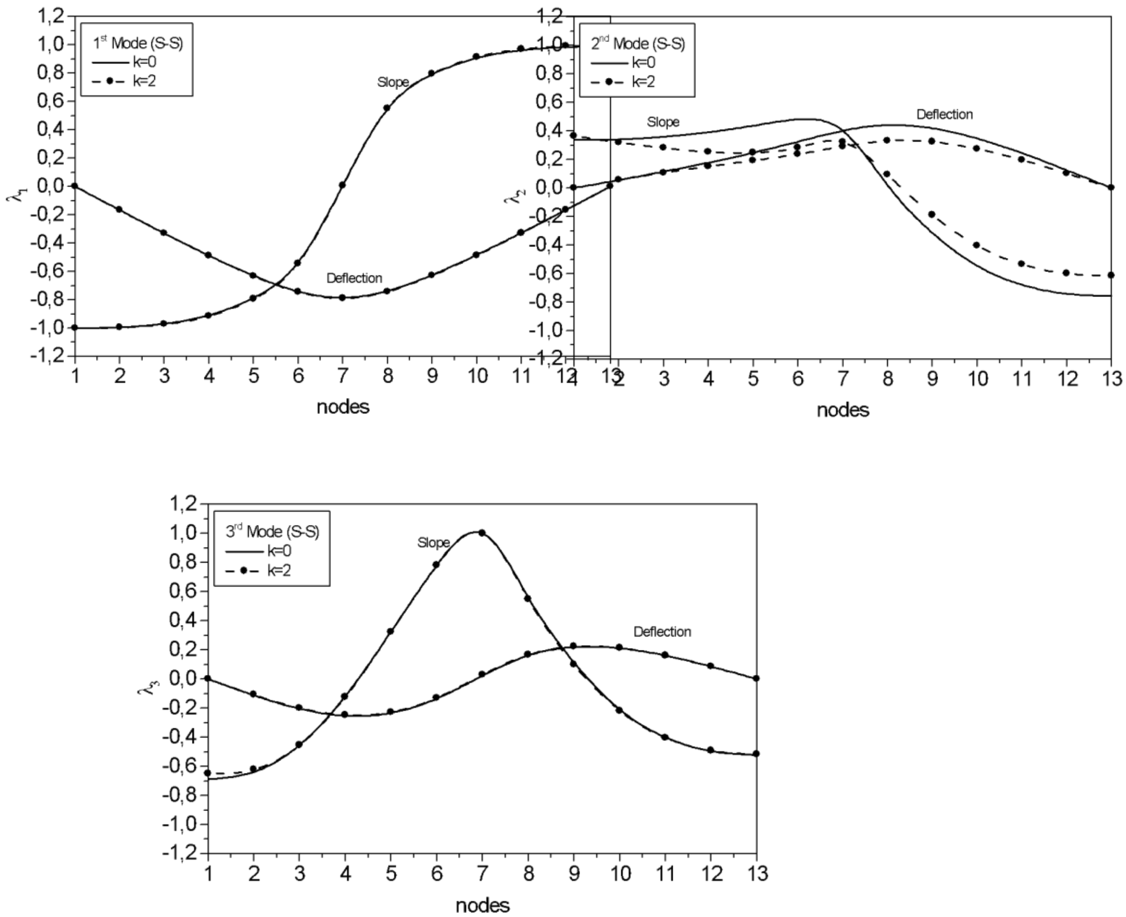


Fig. 3. The variation of mode shapes for simply-supported tapered FGM beam with respect to nodes (L=2, α=0,5, Eratio = 0,5, ρratio=1).

Figure 3 illustrates the mode shapes (slope and deflection) for a simply-supported tapered FGM beam with parameters L=2, α=0,5, $E_{ratio}$ = 0,5, and $\rho_{ratio}$=1, for material non-homogeneity parameters k = 0 and k = 2. It can be observed that the variations in material distribution, as represented by the changes in k, have a significant impact on the second mode shape. In contrast, the first and third mode shapes exhibit minimal sensitivity to these variations.

*Example 2:*

In the second example, the impact of the $E_{ratio}$, tapering parameter, and power law exponent on the first three frequencies of a doubly tapered symmetric clamped beam made from homogeneous material and Functionally Graded Materials (FGMs) is analyzed. The doubly tapered beam exhibits a linear height variation from the clamped end to the center and from the center to the other clamped end. The tapering parameters α considered for non-uniform beams are 0.5 to 0.66.

Table IV and Figure 4 illustrate the variation in fundamental frequency for different beam lengths of a clamped-clamped FGM tapered beam as a function of the power exponent. The results reveal that an increase in the power exponent leads to an increase in frequency when the $E_{ratio}$ is less than 1 ($E_{ratio}$ < 1), a decrease in frequency when the $E_{ratio}$ is greater than 1 ($E_{ratio}$ > 1), and no change in frequency when the $E_{ratio}$ equals 1. This is because an increase in the power law index decreases the value of the elasticity modulus, making FG tapered beams more flexible, and therefore increasing the frequencies. The deflection is proportional to the power law index. Additionally, it can be observed that for a constant power law index and $E_{ratio}$, as the length of the beam decreases, the frequency values also decrease.

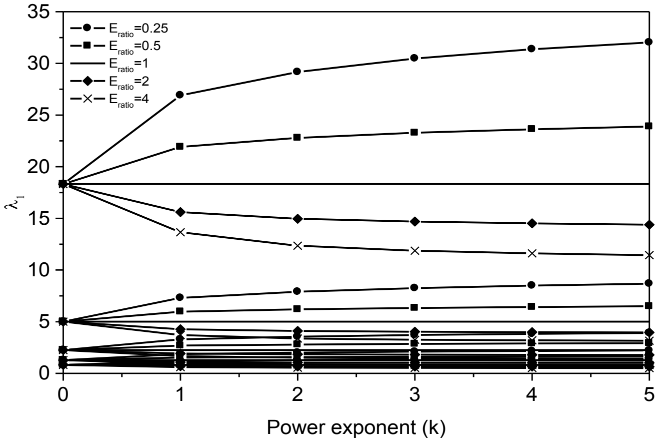


Fig. 4. The variation of the fundamental frequency λ1 for clamped-clamped FGM tapered beam (α=0.66) with power law index k

The effect of modulus ratios on the fundamental frequency for different beam lengths is illustrated in Fig. 5. The results show a significant decrease in the fundamental frequency with an increase in the modulus ratio for all power exponents considered. In contrast, isotropic tapered beams exhibit no significant changes in frequency with varying modulus ratios. Furthermore, for a constant $E_{ratio}$ and a fixed power law index k, the fundamental frequency for a beam length of L=2 m is notably higher than for lengths of L=4, 6, 8, and 10m. This variation is attributed to changes in the geometrical conditions, which subsequently impact the beam's mass and rigidity.

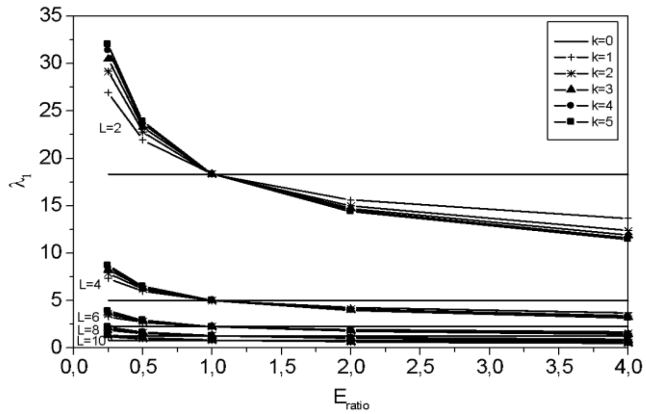


Fig. 5. The variation of the fundamental frequency (λ1) with Young's modulus ratio (α=0,5)

TABLE IV: THE FIRST FREQUENCY PARAMETERS $\lambda_1$ OF C-C TAPERED BEAM FOR DIFFERENT MATERIAL DISTRIBUTION ($E_{ratio} = E_u/E_l$, $\rho_{ratio} = \rho_u/\rho_l = 1$).

| L | $E_{ratio}$ | α | k=0 | k=1 | k=2 | k=3 | k=4 | k=5 |
|---|---|---|---|---|---|---|---|---|
| 2 | 0.25 | 0.5 | 18.3138 | 26.9048 | 29.1657 | 30.4734 | 31.3659 | 32.0241 |
| | | 0.66 | 24.4405 | 36.2696 | 39.4201 | 41.1972 | 42.3838 | 43.2439 |
| | 0.5 | 0.5 | 18.3138 | 21.9185 | 22.7869 | 23.2807 | 23.6255 | 23.8863 |
| | | 0.66 | 24.4405 | 29.3783 | 30.6193 | 31.3034 | 31.7666 | 32.1090 |
| | 1 | 0.5 | 18.3138 | 18.3138 | 18.3138 | 18.3138 | 18.3138 | 18.3138 |
| | | 0.66 | 24.4405 | 24.4405 | 24.4405 | 24.4405 | 24.4405 | 24.4405 |
| | 2 | 0.5 | 18.3138 | 15.6100 | 14.9631 | 14.6896 | 14.5182 | 14.3882 |
| | | 0.66 | 24.4405 | 20.8498 | 19.9087 | 19.4890 | 19.2296 | 19.0408 |
| | 4 | 0.5 | 18.3138 | 13.6541 | 12.3575 | 11.8720 | 11.6164 | 11.4422 |
| | | 0.66 | 24.4405 | 18.3425 | 16.5368 | 15.7951 | 15.3800 | 15.0958 |
| 6 | 0.25 | 0.5 | 2.2549 | 3.2894 | 3.5553 | 3.7127 | 3.8223 | 3.9044 |
| | | 0.66 | 3.3119 | 4.8204 | 5.2133 | 5.4463 | 5.6086 | 5.7300 |
| | 0.5 | 0.5 | 2.2549 | 2.6912 | 2.7908 | 2.8491 | 2.8910 | 2.9234 |
| | | 0.66 | 3.3119 | 3.9463 | 4.0941 | 4.1809 | 4.2432 | 4.2912 |
| | 1 | 0.5 | 2.2549 | 2.2549 | 2.2549 | 2.2549 | 2.2549 | 2.2549 |
| | | 0.66 | 3.3119 | 3.3119 | 3.3119 | 3.3119 | 3.3119 | 3.3119 |
| | 2 | 0.5 | 2.2549 | 1.9202 | 1.8470 | 1.8177 | 1.7991 | 1.7843 |
| | | 0.66 | 3.3119 | 2.8276 | 2.7195 | 2.6753 | 2.6468 | 2.6243 |
| | 4 | 0.5 | 2.2549 | 1.6732 | 1.5206 | 1.4681 | 1.4422 | 1.4246 |
| | | 0.66 | 3.3119 | 2.4719 | 2.2492 | 2.1702 | 2.1299 | 2.1020 |
| 10 | 0.25 | 0.5 | 0.8187 | 1.1940 | 1.2902 | 1.3472 | 1.3870 | 1.4168 |
| | | 0.66 | 1.2119 | 1.7627 | 1.9054 | 1.9903 | 2.0496 | 2.0941 |
| | 0.5 | 0.5 | 0.8187 | 0.9771 | 1.0130 | 1.0341 | 1.0493 | 1.0611 |
| | | 0.66 | 1.2119 | 1.4437 | 1.4972 | 1.5287 | 1.5514 | 1.5690 |
| | 1 | 0.5 | 0.8187 | 0.8187 | 0.8187 | 0.8187 | 0.8187 | 0.8187 |
| | | 0.66 | 1.2119 | 1.2119 | 1.2119 | 1.2119 | 1.2119 | 1.2119 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 0.5 | 0.8187 | 0.6972 | 0.6707 | 0.6602 | 0.6535 | 0.6482 |
| | 0.66 | 1.2119 | 1.0347 | 0.9956 | 0.9798 | 0.9695 | 0.9613 |
| 4 | 0.5 | 0.8187 | 0.6073 | 0.5521 | 0.5332 | 0.5240 | 0.5176 |
| | 0.66 | 1.2119 | 0.9042 | 0.8234 | 0.7950 | 0.7807 | 0.7707 |

## V. Conclusion

In this study, a novel finite element method is used to perform dynamic analysis of tapered functionally graded beams. The uniqueness of this approach lies in the division of the non-uniform beam into segments with uniform cross-sections. Based on the above study, the following conclusions can be drawn:

- Natural frequencies increase with a higher power exponent when $E_{ratio}$ is less than 1, and decrease with a higher power exponent when $E_{ratio}$ is greater than 1. There is no significant change in frequencies for an isotropic beam or when $E_{ratio}$ is equal to 1.
- The tapering ratio has a more significant effect on the third frequency compared to the first and second frequencies.
- The variation in material distribution, as represented by changes in k, has a significant impact on the second mode shape. In contrast, the first and third mode shapes exhibit minimal sensitivity to these variations.
- The frequencies of the C-C beam are higher than those of the simply-supported boundary condition.

The results presented in this paper will be a valuable point of reference for future studies investigating the static behavior of functionally graded tapered I-beams.

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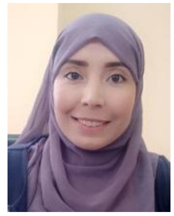

**Dr. Hassina Ziou**

Born in Algeria on January 3, 1991, Dr. Hassina Ziou received her PhD degree in Mechanical Engineering from the University of Mohamed Khider, Biskra, Algeria, in 2017. Her doctoral research focused on advanced studies in composite materials, functionally graded materials (FGMs), and structural dynamics.

She is currently a Senior Researcher at the National Centre for Studies and Integrated Research on Building (CNERIB) in Algeria. Her research spans various fields, including static and vibrational analysis of plates, shells, and beams, nanomechanics, and finite element modeling. She has authored several publications on the mechanical behavior of functionally graded materials (FGMs) and structural dynamics, such as: *Modal Behavior of Longitudinally Perforated Nanobeams*, *The Influence of Various Parameters on the Bending Analysis of Functionally Graded Nanobeams*, and *A Higher-Order Shear Deformation Theory for the Static Bending Analysis of Functionally Graded Beams*. These works highlight her expertise in these areas.

**Dr. Ziou** has received recognition for her contributions to the field. She continues to advance her research on composite materials and mechanical engineering, while also contributing to major publications in the field.