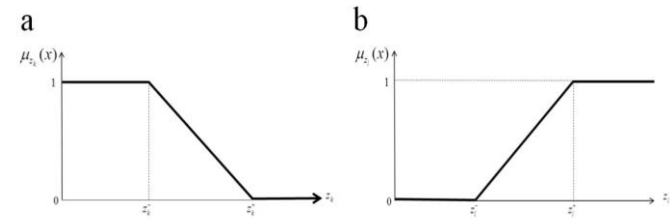

Fig. 3. Membership Function for (a) Min z_k and (b) Max z_l .

In completing the linear programming (10)-(13) not all objective functions simultaneously achieve optimum fuzzy with the constraints provided. So in practice decision makers usually choose Pareto optimal solution as a final decision based on the degree of satisfaction (or level of fuzzy aspiration) at each goal. Pareto optimal solution for linear programming (10)-(13) is defined as follows:

Definition 1. *Visible solution* x^* *on a linear programming (10)-(13) called pareto optimal solution if and only if there is no other visible solution* x^o *so for each* $j,\left(j=1,2,\cdots,k\right)$ *apply* $\mu\left(z_j\left(x^o\right)\right)\geq\mu\left(z_j\left(x^*\right)\right)$ *and* $\mu\left(z_j\left(x^o\right)\right)\neq\mu\left(z_j\left(x^*\right)\right)$ *for at least one* $j,\left(j=1,2,\cdots,k\right)$.

IV. Solving Fuzzy Multi-Objective Supplier Selection Problem

There are three models that can be used to solve the fuzzy multi-objective suppliers selection problem such as:

A. The Symmetric Model

This model was developed by Zimmerman [1]. In this model the objective function is considered to have the same importance. The model formulation is as follows:

Determine $\boldsymbol{x}=\left[x_1,x_2\cdots,x_n\right]^T$ that meets

Max λ

subject to:

$$\begin{aligned}&\lambda\leq f_{\mu zj}\left(x\right),\ j=1,2,\cdots,q\\&g_r\left(x\right)\leq b_r,\ r=1,2,\cdots,h\\&x_i\geq 0,\ i=1,2,\cdots,n\\&0\leq\lambda\leq 1\end{aligned}\qquad(16)$$

B. The Weighted Addtive Model

This model was developed by [2]. This model is a nonsimetris model because for each objective function is considered to have different importance. The model formulation is as follows:

Determine $\boldsymbol{x}=\left[x_1,x_2\cdots,x_n\right]^T$ that meets

Max $\sum_{j=1}^{q}w_j\lambda_j$

subject to:

$$\begin{aligned}&\lambda_j\leq f_{\mu zj}\left(x\right),\ j=1,2,\cdots,q\\&g_r\left(x\right)\leq b_r,\ r=1,2,\cdots,h\\&\lambda_j\in\left[0,1\right],\ j=1,2,\cdots,q\\&\sum_{j=1}^{q}w_j=1,w_j\geq 0\\&x_i\geq 0,\ i=1,2,\cdots,n\end{aligned}\qquad(17)$$

where w is the weight of the objective function.

C. The Weighted Max-Min Model

This model was developed by [2] to improve the weighted additive model that does not guarantee consistency between the achievement level of the objective function and the functional weight. The model formulation is as follows:

Determine $\boldsymbol{x}=\left[x_1,x_2\cdots,x_n\right]^T$ that meets

Max λ

subject to:

$$\begin{aligned}&w_j\lambda\leq f_{\mu zj}\left(x\right),\ j=1,2,\cdots,q\\&g_r\left(x\right)\leq b_r,\ r=1,2,\cdots,h\\&\sum_{j=1}^{q}w_j=1,w_j\geq 0\\&x_i,\lambda\geq 0,\ i=1,2,\cdots,n\end{aligned}\qquad(18)$$

where w is the weight of the objective function.

Linear programming (16), (17) and (18) are deterministic linear programming or ordinary linear programming, so they can be solved by the simplex method. For more details about the transformation of the fuzzy multi-objecttive linear programming into a deterministic single-objective linear program can be seen in [1]-[2]. By completing (16), (17) and (18) obtained the optimal solution λ^* and x^*. The optimal solution x^* x^* is the optimal solution of linear programming (10)-(13) and λ^* is the membership degree for $z_j\left(x^*\right),\ j=1,2,\cdots,q$. The following two theorems will be shown to ensure that the optimal solution obtained from the weighted max-min model (18) is the optimal solution of the linear programming (10)-(13) and with the same analogy can be developed for the other two models.

Theorem 1. *If* $x^*\in X$ *is the unique optimal solution of weighted max-min model (18) for some* $w=\left(w_1,\cdots,w_q\right)>0$, *then* x^* *is the Pareto optimal solution of the linear programming (10)-(13).*

Proof. Is known that x^* is unique optimal solution of weighted max-min problem (18) for some $w=\left(w_1,\cdots,w_q\right)>0$, based on (18) it means x^* the only member of X that meets

$x^* = \max \min_{j=1,2,\cdots,q} \frac{f_{\mu zj}(x)}{w_j}$. If x^* is not Pareto optimal solution of the linear programming (10)-(13), then there exists $x^o \in X$ such that $f_{\mu zi}(x^o) < f_{\mu zi}(x^*)$ for some i and $f_{\mu zj}(x^o) \le f_{\mu zj}(x^*)$, $j = 1, 2, \cdots q; j \ne i$.

So $\frac{f_{\mu zi}(x^o)}{w_j} < \frac{f_{\mu zi}(x^*)}{w_j}$ for some i and $\frac{f_{\mu zi}(x^o)}{w_j} \le \frac{f_{\mu zi}(x^*)}{w_j}$, $j = 1, 2, \cdots q; j \ne i$. If the minimum value is taken for $j = 1, 2, \cdots q$ obtained:

$$\min\left\{\frac{f_{\mu zi}(x^o)}{w_j}\right\} \le \min\left\{\frac{f_{\mu zi}(x^*)}{w_j}\right\}.$$

Furthermore, if the maximum value is taken, obtained:

$$\max\left\{\min\left\{\frac{f_{\mu zi}(x^o)}{w_j}\right\}\right\} \le \max\left\{\min\left\{\frac{f_{\mu zi}(x^*)}{w_j}\right\}\right\}.$$

This contradicts the statement x^* is the single optimal solution of the weighted max-min model (18). Hence, x^* is the Pareto optimal solution of the linear programming (10)-(13).

■

Theorem 2. *If $x^* \in X$ is the Pareto optimal solution of the linear programming (10)-(13), then x^* is the optimal solution of the weighted max-min model (18) for some* $w = (w_1, \cdots, w_q) > 0$.

Proof. Is known that $x^* \in X$ is Pareto optimal solution of the linear programming (10)-(13), then by Definition 1, it,s means that $f_{\mu zj}(x^*) \le f_{\mu zj}(x^o), x^o \in X$. Next, we choose a weight $w^* = (w_1^*, \cdots, w_q^*) > 0$ such that $\frac{f_{\mu zi}(x^*)}{w_j^*} = \lambda,\ j = 1, 2, \cdots q$.

For this weight, if x^* is not the optimal solution of the weighted max-min model (18), then there exists $x^o \in X$ so

$$\max\left\{\min\left\{\frac{f_{\mu zi}(x^o)}{w_j}\right\}\right\} \le \max\left\{\min\left\{\frac{f_{\mu zi}(x^*)}{w_j}\right\}\right\}.$$

As a consequence, there exists $x^o \in X$ so $f_{\mu zj}(x^o) \le f_{\mu zj}(x^*)$, $j = 1, 2, \cdots q$. This is a contradiction with x^* x^* as Pareto optimal solution of linear programming (10)-(13). Hence, x^* is the optimal solution of the weighted max-min model (18).

■

The value of the new membership function and the optimal level of achievement (λ^*) can exceed the current unity when $w_j < 1$. However, the actual level of achievement for each objective function may never exceed unity. This model produces the optimal solution in the visible region so that the ratio of achievement levels of the membership function is as close as possible to the ratio of objective function weights [6].

For getting the weight or importance level between each fuzzy goal of the decision maker is a very important initial process to complete this model. For determine weights of the objective function in this paper are used Analytic Hierarchy Process (AHP).

V. Analytic Hierarchy Process (AHP) and Algorithm for Solving Fuzzy Multi-Objective Supplier Selection Problem

A. Analytic Hierarchy Process (AHP)

Algorithm for determine weights of objective functions are as follows:

1. Create a pairwise comparison matrix of the importance of each objective function desired by the decision maker.
2. If the comparison matrix of a consistently perfect decision maker then the next step is the determine vector $\boldsymbol{w} = [w_1\ w_2\ \ldots\ w_n]$ that is nontrivial solution of the system of n equations $A\boldsymbol{w}^T = \Delta \boldsymbol{w}^T$ with Δ is an unknown number and is an unknown n-dimension column matrix.
3. If the matrix of comparison from the decision maker is not consistent then the matrix $\boldsymbol{w}$ estimated based $\boldsymbol{w}_{maks}$.
4. Conduct consistency test on comparison matrices.

For more details about algorithm to determine weights of objective function with AHP, it can be seen in [10]-[9].

B. Algorithm to solving the fuzzy multi-objective supplier selection.

Step **1**: Declare the problem of supplier selection as a multi-objective linear programming based on the goals to be achieved and the constraints faced.
Step **2**: Determine minimum and maximum individual for each objective function based on the existing constraints.
Step **3**: Ask the decision maker about the goal and the value of the constraints that want to be achieved along with the value of tolerance and leniency for the goal. Goal with such leeway is called fuzzy goal.
Step **4**: Determine membership function for fuzzy objective function based on fuzzy goal given by decision maker.
Step **5**: If all the objective functions have the same importance then the fuzzy multi-objective supplier selection problem is directly transformed into a deterministic single-objective linear programming using the equation (16) and proceed directly to Step 8.
Step **6**: If each objective function has a different level of importance then asks the decision maker to determine the weight of each fuzzy goal by using the Analytic Hierarchy Process approach.
Step **7**: Transform the supplier selection problem of a fuzzy multi-objective linear program becomes a single-objective

deterministic linear programming problem as in equation (17) or equation (18).

***Step* 8**: Determine optimal solution x^* by solving the deterministic single-objective linear programming problem using Simplex method.

This algorithm is illustrated in the following numerical example.

VI. Numerical Example

For supplying a new product to the market, it is assumed that there are three suppliers to choose from by a decision maker. All three suppliers have different capabilities and capacities in providing the new product. Criteria for purchasing products from these three suppliers are the cost of purchase, quality and service.

Goal to be achieved by the decision makers of the purchase of new products is to minimize the cost of purchase, maximize the quality of products obtained and maximize the level of service that will be obtained from suppliers. Supplier capacity constraints are also considered in the model. It is assumed that the information on supplier performance on the above criteria is not accurately known. The estimated value of cost, quality and service levels and supplier constraints are presented in Table 1. Market demand to be met is 1000 tons.

The model formulation of supplier selection problem above is as follows:

Min $Z_1 = 13x_1 + 11.5x_2 + 15x_3$

Max $Z_2 = 0.8x_1 + 0.7x_2 + 0.9x_3$

Max $Z_3 = 0.85x_1 + 0.75x_2 + 0.8x_3$

subject to:

$x_1 + x_2 + x_3 = 1000$

$x_1 \le 700, \quad x_2 \le 600, \quad x_3 \le 500$

$x_i \ge 0, i = 1, 2, 3$

Three objective functions Z_1, Z_2 and Z_3 cost, quality and service, respectively, and x_i is the number of units purchased from the ith supplier.

Table II presents the individual maximums and minimums of the three objective functions, the maximum and minimum individual values then became fuzzy goals for each objective function. Equations (19), (20) and (21) show the membership function of the three fuzzy goals.

TABLE I: Suppliers' Quantitative Information

	Price (million rupiah/ton)	Quality (%)	Service (%)	Capacity (ton)
Supplier 1	13	80	85	700
Supplier 2	11.5	70	75	600
Supplier 3	15	95	80	500

Furthermore, the problem of supplier selection is solved by three different methods, weightless method (symmetric), weighted additive method and weighted max-min method whose algorithm has been formed in the previous section.

TABLE II: The Data Set for Membership Functions

Objective Function	$\mu = 0$	$\mu = 1$	$\mu = 0$
Z_1 (Purchase Cost)	-	12100	14000
Z_2 (Quality Level)	740	875	-
Z_3 (Service Level)	770	835	-

For $Z_1 = 13x_1 + 11.5x_2 + 15x_3$ (Purchase Cost) :

$$\mu_{z1}(x) = \begin{cases} 1 & ; \quad z_1 \le 12100 \\ \dfrac{14000 - z_1}{1900} & ; \quad 12000 \le z_1 \le 14000 \\ 0 & ; \quad z_1 \ge 14000 \end{cases} \tag{19}$$

For $Z_2 = 0.8x_1 + 0.7x_2 + 0.9x_3$ (Quality Level)

$$\mu_{z2}(x) = \begin{cases} 1 & ; \quad z_2 \ge 875 \\ \dfrac{z_2 - 740}{135} & ; \quad 740 \le z_2 \le 875 \\ 0 & ; \quad z_2 \le 740 \end{cases} \tag{20}$$

For $Z_3 = 0.85x_1 + 0.75x_2 + 0.8x_3$ (Service Level):

$$\mu_{z3}(x) = \begin{cases} 1 & ; \quad z_3 \ge 835 \\ \dfrac{z_3 - 770}{65} & ; \quad 770 \le z_3 \le 835 \\ 0 & ; \quad z_3 \le 770 \end{cases} \tag{21}$$

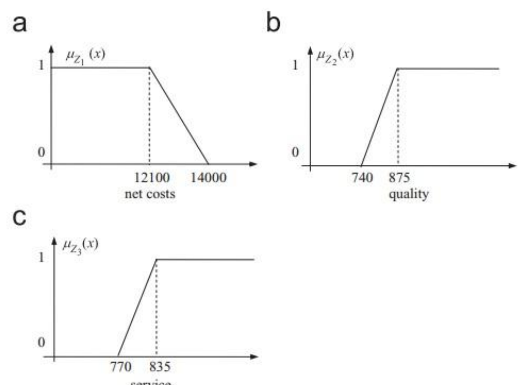

Fig. 4. Membership Function (a) Purchase Cost, (b) Quality Level, and (c) Service Level.

For Symmetrical Model:

Deteremine $[x_1, x_2, x_3]^T$ that meets

Max λ

subject to:

$$\lambda \le \frac{14.000 - (13x_1 + 11.5x_2 + 15x_3)}{1900}$$

$$\lambda \le \frac{(0.8x_1 + 0.7x_2 + 0.9x_3) - 770}{135}$$

$$\lambda \le \frac{(0.85x_1 + 0.75x_2 + 0.8x_3) - 740}{65}$$

$$x_1 + x_2 + x_3 = 1000$$

$$x_1 \le 700, \quad x_2 \le 600, \quad x_3 \le 500$$

$$x_i \ge 0, i = 1,2,3$$

$$\lambda \in [0,1]$$

For Weighted Additive Model:

Determine $[x_1, x_2, x_3]^T$ that meets

Max $0.63\lambda_1 + 0.11\lambda_2 + 0.26\lambda_3$

subject to:

$$\lambda_1 \le \frac{14.000 - (13x_1 + 11.5x_2 + 15x_3)}{1900}$$

$$\lambda_2 \le \frac{(0.8x_1 + 0.7x_2 + 0.9x_3) - 770}{135}$$

$$\lambda_3 \le \frac{(0.85x_1 + 0.75x_2 + 0.8x_3) - 740}{65}$$

$$x_1 + x_2 + x_3 = 1000$$

$$x_1 \le 700, \quad x_2 \le 600, \quad x_3 \le 500$$

$$x_i \ge 0, i = 1,2,3$$

$$\lambda_j \in [0,1], j = 1,2,3$$

For Weighted Max-Min Model:

Determine $[x_1, x_2, x_3]^T$ that meets

Max λ

subject to:

$$0.63\lambda \le \frac{14.000 - (13x_1 + 11.5x_2 + 15x_3)}{1900}$$

$$0.11\lambda \le \frac{(0.8x_1 + 0.7x_2 + 0.9x_3) - 770}{135}$$

$$0.26\lambda \le \frac{(0.85x_1 + 0.75x_2 + 0.8x_3) - 740}{65}$$

$$x_1 + x_2 + x_3 = 1000$$

$$x_1 \le 700, \quad x_2 \le 600, \quad x_3 \le 500$$

$$x_1, x_2, x_3, \lambda \ge 0$$

With the help of POM for Windows software, obtained the results as presented in Table III.

Based on Table III, on a weightless approach or a symmetrical approach, there is no difference of importance between the three criteria so the objective function having the same weight, resulting in the level of attainment for all objective functions is same, $\mu_{Z1} = \mu_{Z2} = \mu_{Z3} = 0.9$. Furthermore, Table III shows that the weighted additive model is unacceptable because the achievement levels are not corresponding to the weight of the objective function. The achieved level of the first objective function is lower than the achieved level of the second objective function even though the weight of first objective function is greater than the weight of the second objective function. Comparing the solution obtained by the weighted max-min approach, it can be seen that the proposed model succeeded in finding the optimal solution such that the ratio of the achievement level of the objective function is equal to the ratio of the weight of its objective function and the solution more consistent than the other approach solution with the decision maker's preference or expectation. In other word $(\mu_1 > \mu_3 > \mu_2)$ agrees with $(w_1 > w_3 > w_2)$.

TABLE III: THE CALCULATIONS RESULTS OF NUMERICAL EXAMPLE WITH THREE DIFFERENT APPROACHES

	Method 1	Method 2	Method 3
Z_1	12380	13600	13836
Z_2	760	845	863
Z_3	793	835	829
x_1	386	700	582
x_2	528	0	0
x_3	86	300	418
μ_1	0.85	0.21	0.9
μ_2	0.15	0.77	0.9
μ_3	0.35	1	0.9

where Method 1 is The Weighted Max-Min Method, Method 2 is The Weighted Additive Method and Method 3 is The Weightless Method (Symmetric Method).

VII. Conclusion

Based on the application of the three methods in the numerical example it can be concluded that the weighted max-min method is the best method to solving the fuzzy multi-objective supplier selection problem.

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